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ELASTIC MODULII AND THEIR RELATIONSHIP BY CONSIDERING ANY ARBITRARY ANGLE

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ABSTRACT

In this paper, we obtained a relationship among Young Modulus of Elasticity E, *Bulk Modulus of Elasticity* K *and Rigid Modulus of Elasticity* G *by considering any arbitrary angle.*

Keywords and Phrases: Young Modulus of Elasticity; Bulk Modulus of Elasticity; Rigid Modulus of Elasticity; Relation between elastic moduli.

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1. INTRODUCTION

Hooke's Law

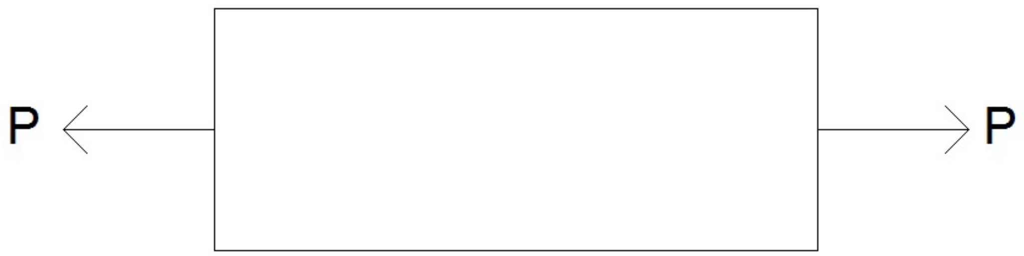
Stress produced in a body is directly proportional to strain in the body, within the elastic limit.

$$\frac{\text{stress}}{\text{strain}} = \text{Constant of elasticity}$$

Now depending upon the type of stress and strain produced in the body, constant of elasticity is of the following three types:

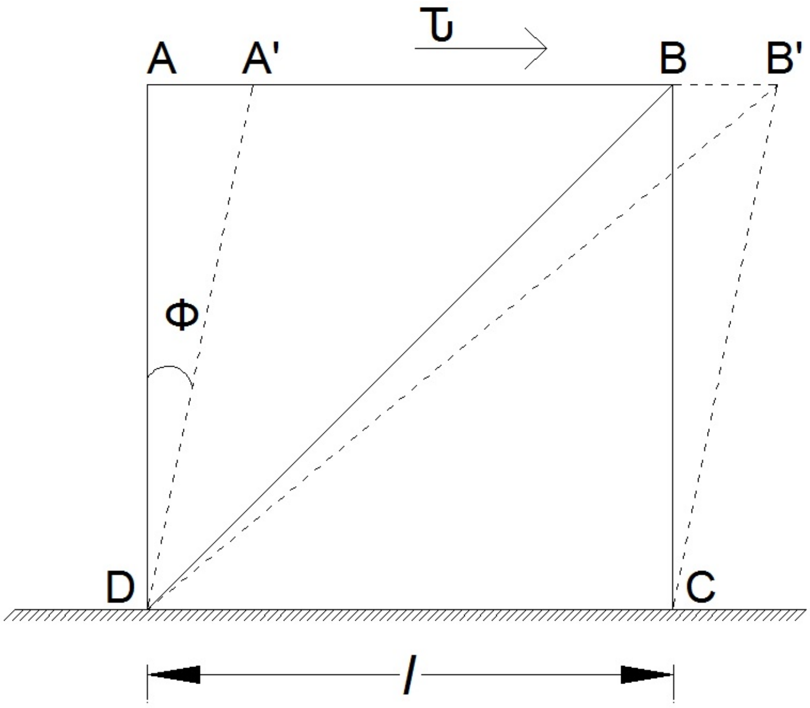
(i) Modulus of Elasticity (E)

$$E = \frac{\text{Normal stress}}{\text{Normal strain}} = \frac{\sigma_n}{e} \tag{1.1}$$



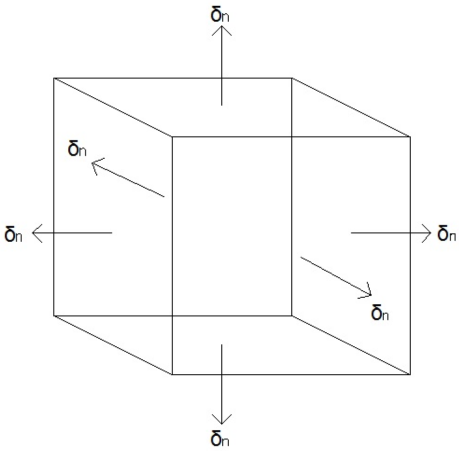
(ii) Modulus of Rigidity (G)

$$G = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{e_s} = \frac{\tau}{\tan\phi} \tag{1.2}$$



(iii) Bulk modulus (K)

$$K = \frac{\text{Hydrostatic stress}}{\text{Volumetric strain}} = \frac{\sigma_n}{e_v} \tag{1.3}$$



2. MAIN RESULT

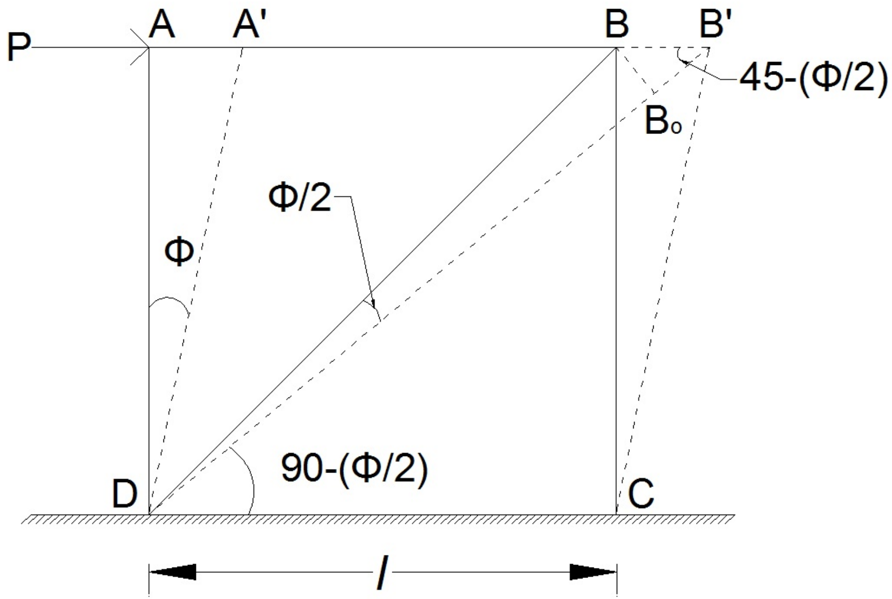
The relationship among Young Modulus of Elasticity among E, Bulk Modulus of Elasticity K and Rigid Modulus of Elasticity G is given by

$$E = \frac{9KG}{G + 3\sqrt{2}\,K\cos\left(\frac{\pi}{4} - \frac{\phi}{2}\right)} \tag{2.1}$$

where ϕ is the angle through which face AD tilted from its original position.

3. DERIVATION

Consider a two dimensional body subjected to shear stress as its lower surface is fixed and force is applied on the upper surface. Due to produced shear stress τ the length of the diagonal DB will increase and length of diagonal AC will increase. The surface AB will shift rightward to $A'B'$ as shown in figure.



From triangle ADA'

$$\tan\phi = \frac{AA'}{AD} = \frac{AA'}{\ell}$$

$$AA' = \ell\tan\phi = BB' \tag{3.1}$$

$$\text{Strain produced in diagonal DB} = \frac{DB' - DB}{DB} = \frac{B_0 B'}{\sqrt{2}\,\ell} \tag{3.2}$$

From triangle $B_0 BB'$

$$\cos\left(\frac{\pi}{4} - \frac{\phi}{2}\right) = \frac{B_0 B'}{BB'}$$

$$B_0 B' = BB'\cos\left(\frac{\pi}{4} - \frac{\phi}{2}\right) \tag{3.3}$$

From (3.2) and (3.3), we get

$$\text{Strain produced in diagonal DB} = \frac{BB'\cos\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}{\sqrt{2}\,\ell} \tag{3.4}$$

From (3.1) and (3.4), we have

$$\text{Strain produced in diagonal DB} = \frac{\tan\phi\cos\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}{\sqrt{2}} \tag{3.5}$$

Linear strain produced in diagonal DB

$$\frac{\tau}{E} + \nu\frac{\tau}{E} = \frac{\tau}{E}(1+\nu) \tag{3.6}$$

From (3.5) and (3.6), we get

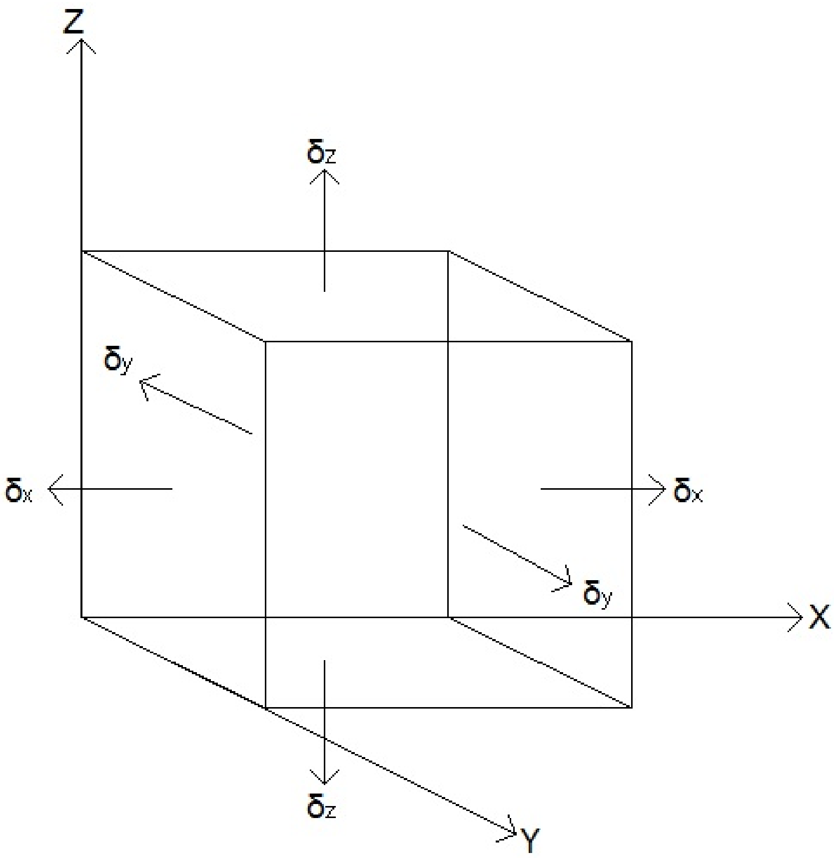
$$\frac{\tau}{E}(1+\nu) = \frac{\tan\phi\cos\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}{\sqrt{2}}$$

$$E = \frac{\sqrt{2}\,\tau\,(1+\nu)}{\tan\phi\cos\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}$$

From (1.2), we have

$$E = \frac{\sqrt{2}\,G(1+\nu)}{\cos\left(\frac{\pi}{4} - \frac{\phi}{2}\right)} \tag{3.7}$$

Consider a three dimensional body of length ℓ, breadth b and thickness t. Suppose the stress induced in the body along length ℓ be σ_x. Similarly stress induced in the body along breadth b and thickness t be σ_y and σ_z respectively. As we know that strain induced in the direction of force applied is longitudinal or linear strain and strain induced in the perpendicular direction of force applied is lateral strain.



For calculating the linear strain in the particular direction, lateral strain induced in the other direction by the same force is to be subtracted for that particular direction. Strain produced in the length ℓ

$$e_\ell = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$e_b = \frac{\sigma_y}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_z}{E}$$

$$e_t = \frac{\sigma_z}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E}$$

Overall volumetric strain produced is

$$e_V = e_\ell + e_b + e_t = \frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \frac{\sigma_z}{E} - 2\nu\left(\frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)$$

$$e_V = \frac{3\sigma_n}{E} - \frac{6\nu\sigma_n}{E} = \frac{3\sigma_n}{E}(1-2\nu)$$

$$E = \frac{3\sigma_n}{e_V}(1-2\nu)$$

$$E = 3K(1-2\nu) \tag{3.8}$$

Eliminating ν from (3.7) and (3.8), we get the main result (2.1).

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